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Optical model studies for elastic scattering of protons

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Abstract : The angular distribution of protons elastically scattered from ^{12}C, ^{16}O, ^{27}Al, ^{28}Si, ^{40}Ca, ^{58,60,62,64}Ni, ^{63,64,65}Cu, ^{64,66,68}Zn, ^{90}Zr, ^{108}Ag, ^{116}Sn, ^{197}Au, ^{208}Pb at different energies ranging from 28 MeV to 61 4 MeV, have been analyzed in terms of the spherical optical model Independent energy search at each energy and common geometry optical model calculations have been performed An over all satisfactory agreement has been obtained The best fit parameter values of the depth, radius and diffusivity of all the real, imaginary and spin orbit parts of the potential are obtained by the method of least square Systematic trends of variation of different potential depths are investigated. Imperical formulae have been developed for the real potential depths of the proton scattering

Keywords · Optical model potential, sensitivity of potential parameters

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1. Introduction

Elastic scattering of nucleons from nuclei can be described in terms of a local optical model. The optical potential is described in terms of some parameters which show a smooth and sensible variation with projected energy and with the neutron and proton number of the target nucleus. The model does not yield a unique optical potential even though the number of parameters can be varied from 6 to 12. Although by now, nearly half a century passed since the model was born, yet works on the model are still on progress. Different approaches of calculations have subdivided the field. Experimental data are being used for optical model fits for comparing it to the measured quantities and thus values of different parameters are found out. These parameters, in turn, describe physics of the microscopic world. All sorts of projectiles, like light ions and heavy ions, electrons, neutrons, pions *etc.* have been used in the scattering experiments. The interaction process is broadly classified into two groups – the elastic scattering and non-elastic interactions. Attempts have also been taken to study elastic scattering between light nuclei in terms of the optical model. Several such studies were done in the past [1]. Proton elastic scattering angular distributions for few targets (Fe, Pb, Ni) are studied in the framework of the optical model using Woods-Saxon (WS) and squared WS potentials by a few authors [2]. Angular distributions of the differential cross section of alpha elastic scattering by different target nuclei have also been studied by other authors [3,4]. Moreover, in the case of scattering of alpha particles from light nuclei, the WS type potentials have proven quite inadequate, while the squared WS type potentials have emerged remarkably successful [3].

The present work is an effort of analysis of scattering experiments using optical model. It aims at studying elastic scattering of protons at different energies ranging from 28 MeV to 61.4 MeV on 20 nuclei, *e.g.*, ^{12}C, ^{16}O, ^{27}Al, ^{28}Si, ^{40}Ca, ^{58,60,62,64}Ni, ^{63,64,65}Cu, ^{64,66,68}Zn, ^{90}Zr, ^{108}Ag, ^{116}Sn, ^{197}Au and ^{208}Pb. Now, the reason for the

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choice of targets, projectile and the energy range so made, are as follows :

(a) Many data are available for these nuclei. Moreover, most of the nuclei are spherical and are therefore, well described by the simple optical model potential with no strong coupling to low lying collective states.

(b) The energy range 28–61.4 MeV studied for proton scattering lie well within the accepted limits, *e.g.* 1–160 MeV of validity of the microscopic optical model [5].

(c) Protons appear to have several possible advantages over composite particles; furthermore, the higher penetrability of protons in nuclear matter allows them to probe the nuclear interior. Finally, proton scattering is more suitable for more fundamental analysis.

2. Optical model calculations

The potential considered in the calculation can be described as follows :

(a) The real part of the potential is described by a volume potential of the Woods-Saxon form.

(b) The imaginary part is a surface potential having Woods-Saxon derivative form.

(c) The spin-orbit form was chosen as is given by the following formulae :

$$V'_{s0} = (v_{s0} + iw_{s0})(1/r)\left[4\exp\{(r-r_w)/a_w\}\right]/$$

$$\left[1+\exp\{(r-r_w)/a_w\}\right]^2 \quad \mathbf{l.\,S}$$

Here, v_{s0} and w_{s0} are strengths of the real and imaginary spin-orbit potentials.

$r_w = R_w A^{1/3}$ is the radius and a_w measures the surface diffuseness.

It is well known that the optical model potential should be non-local in form, *i.e.*, $V(r)\ y(r)$ should be replaced by $\grave{o}V(\boldsymbol{r}, \boldsymbol{r}')\ y(\boldsymbol{r}')d\boldsymbol{r}'$, of course this term is equivalent to a local potential with energy-dependent coefficients when certain approximations are made about it. For calculating the differential scattering of protons, it was necessary to solve the Schrödinger equation

$$\nabla^2\psi + (2\mu/h^2)(E-V)\psi = 0,$$

m = the reduced mass, E = the energy of the incident particle in the centre of mass system, V = the optical model potential. Particle wave method was applied to solve the equation. For the 1-th partial wave, the corresponding radial equation is given by

$$d^2u_1/dr^2 + \{(2\mu/h^2)(E-V(r)) - 1(1+1)/r^2\}u_1 = 0.$$

This equation was then solved numerically.

2.1. *Parameter sensitivity :*

In optical model formalism, there are nine parameters in total. These are the real, imaginary and spin-orbit potential depths (U, W, V_{s0} respectively), the corresponding radius parameters, R_u, R_w, r_{s0} and the diffuseness parameters A_u, A_w and a_{s0}. These parameters of the model are interrelated. It is impossible to isolate one by ignoring the effects of others of the set. Hence, they must be varied systematically to reproduce the experimental observables. An idea of their sensitivity on the angular distribution is necessary. The present work also attempts for such an analysis. During the analysis of the parameter sensitivity, a single parameter was varied alone at a time while others were kept unaltered. Figure 1a shows that scattering increases with an increase in the depth of the real potential U. A reverse trend is observed in case of the radius parameter R_u. Additionally, in the case of R_u, a lateral shift in the oscillation pattern is also obvious. On the other hand A_u, the real part of the central potential slightly increases the overall cross section values, except a decrease in the region 90°–175° (Figure 1c). In both the cases of variation of W and R_w, cross section decreases with increment of the parameters throughout the angular range without any noticeable effect to the frequency of oscillation. In the last case *i.e.*, for the variation of A_w of the imaginary potential, there is no observable difference (Figure 1f) in the angular distribution except at large angles. However, the value of A_w is correlated with the imaginary depth W.

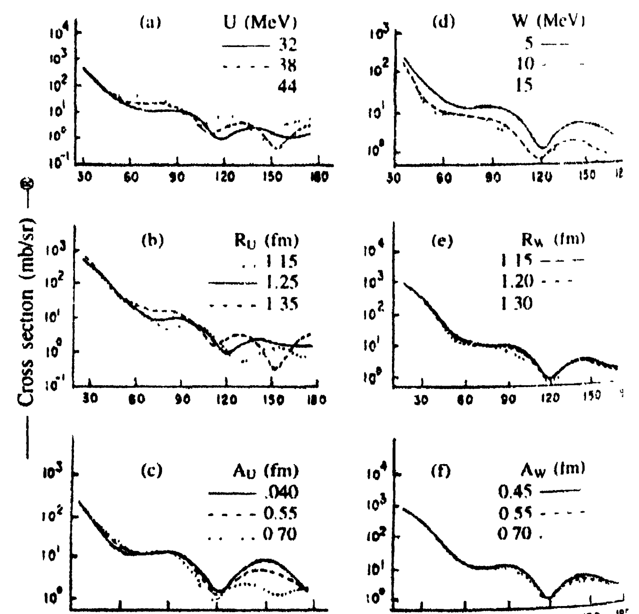

Figure 1. Effect of variation of potential parameters (a) U, (b) r_u, (c) a_u, (d) W, (e) r_W and (f) a_W on angular distribution.

3. Results and discussion

3.1. *Energy variation of optical potential :*

3.1.1. *Real part of the optical model potential :*

The results of the real part of the potential as a function of incident energy for the nuclei $^{63,65}Cu$, ^{58}Ni and ^{68}Zn are shown in Figure 2a. The data have been fitted by the following three straight lines :

(i) $U = 53.3 - 0.55\,E + 27(N-Z)/A + 0.4\,Z/A^{1/3}$ MeV. (1)

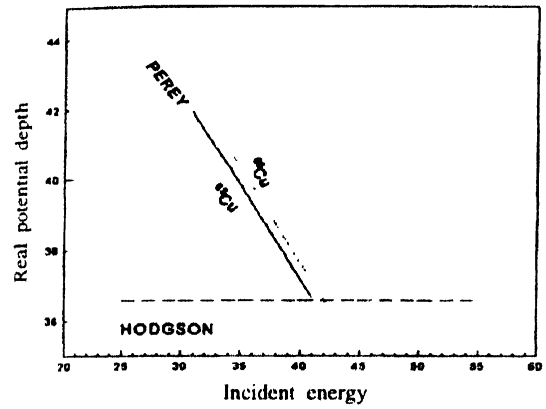

Figure 2a. Energy variation of the real part of the optical model potential

This relation was proposed by Percy and Percy [6] after an extensive analysis of proton elastic scattering in the range 9 22 MeV for medium weight nuclei.

$$\text{(ii)}\quad V = 106.3\,A/\left[R^3(1+\Pi^2 a^2/R^2)\right] + 24\alpha + 0.517\,T(1-\alpha)/\alpha R. \qquad (2)$$

Here, $R = 1.204\,A^{1/3} + 0.305$, $a = (N-Z)/A$ and T is the iso-spin of the nucleus. Hodgson [7] showed that the data of Percy and Percy [6] with $50 < A < 70$ is inconsistent with eq. (1) and a better phenomenological expression for this energy and mass range would be given by eq. (2).

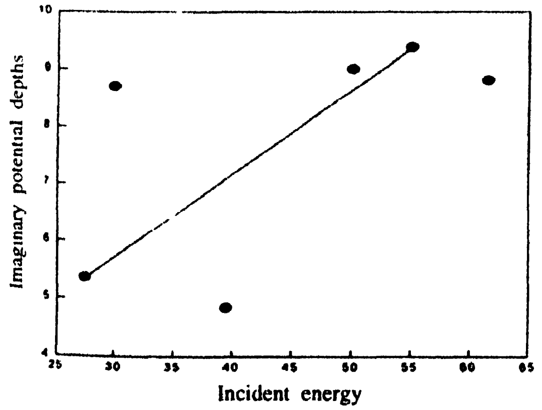

Figure 2b. Energy variation of the imaginary part of the optical model potential.

(iii) $Vp = 54.0 - 0.32\,E$ MeV, $E < 50$ MeV,

a relation proposed by Becchetti and Greenlees [8] for the scattering of protons.

From the Figure 2a it can be observed that the relation proposed by Percy and Percy fits best with the real part of the optical potential obtained in the present work.

3.1.2. *Imaginary part of the optical model potential :*

The results of variation of the imaginary part of the optical potential as a function of energy have been shown in Figure 2b. A linear energy dependence

$$W = 1.17 + 0.15\,E \text{ MeV},\ 5 < E < 40 \text{ MeV} \qquad (3)$$

was found in the work. This relation was proposed by Brieva and Rook [9] after analyzing nucleon-nucleus optical model potential considering nuclear matter approach.

3.2. *Asymmetry variation of optical model potential :*

3.2.1. *Real part of the optical model potential :*

The asymmetry variation of the real part of the optical model potential has been shown in Figure 3a for $E_p = 39.6$ MeV. The real potential depths were corrected for the Coulomb term $0.4\,Z/A^{1/3}$. The straight line (solid) shows the results predicted by the following equation with Percy's values of $U_0 = 36.6$ MeV and $U_1 = 27$ MeV.

$$U - 0.4\,Z/A^{1/3} = U_0 + U_1(N-Z)/A \qquad (4)$$

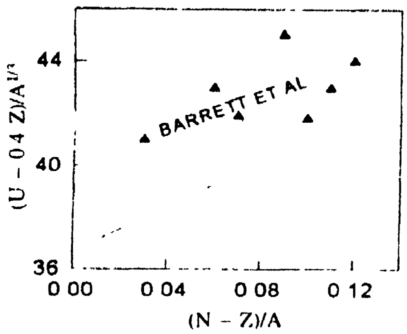

Figure 3a. Real potential depths corrected for the Coulomb term as a function of the nuclear symmetry parameter.

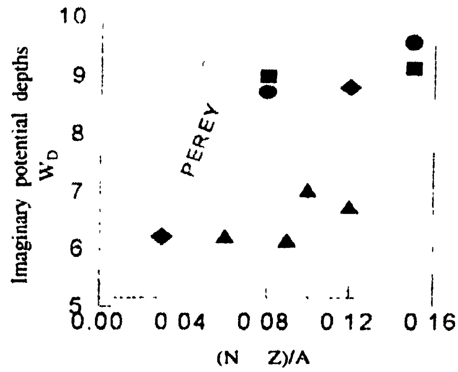

Figure 3b. The imaginary surface potential depths plotted as function of $(N-Z)/A$, showing the straight line corresponding to $W_D = 7 + 50(N-Z)/A$

The dotted curve shown in the figure has been drawn using the values of $U_0 = 39.6$ MeV and $U_1 = 17$ MeV for surface absorption due to Barrett *et al* [10]. It is obvious that Barrett's values suit better with the present observation.

3.2.2. Imaginary part of the optical model potential :

Imaginary surface potential depths have been plotted as a function of $(N-Z)/A$ and $A^{1/3}$ in Figures 3b and 3c respectively. From the Figure 3c, it can be observed that Percy's estimation $W_D = 3A^{1/3}$ is approximately valid.

The line in Figure 3b is $W = 7 + 50\ (N-Z)/A$, as suggested by Percy and Percy also shows a satisfactory fit.

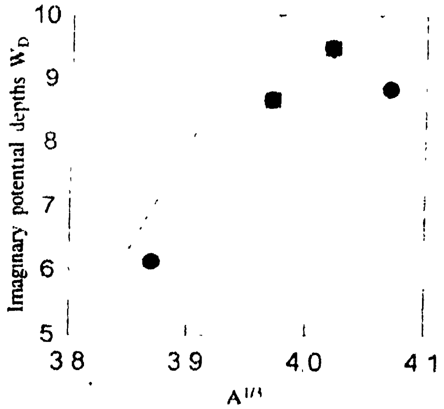

Figure 3c. The imaginary surface potential depths plotted as function of $A^{1/3}$, showing the straight line corresponding to $W_D = 3A^{1/3}$

3.3. Variation of radius and diffuseness parameters :

Presence of any trend of variation of radius and diffuseness parameters has also been searched in the work, but no such trend was found. From the works, it appears that the values of the radius parameters and the diffuseness parameters are largely independent of the incident particle energy and the mass of the target nuclei.

4. Conclusion

The angular distributions of protons elastically scattered from ^{12}C, ^{16}O, ^{27}Al, ^{28}Si, ^{40}Ca, $^{58,60,62,64}Ni$, $^{63,64,65}Cu$, $^{64,66,68}Zn$, ^{90}Zr, ^{108}Ag, ^{116}Sn, ^{197}Au and ^{208}Pb at different energies ranging from 28 MeV to 62 MeV and the polarization data have been analyzed in terms of the spherical optical model and have been depicted in Figures 4, 5.

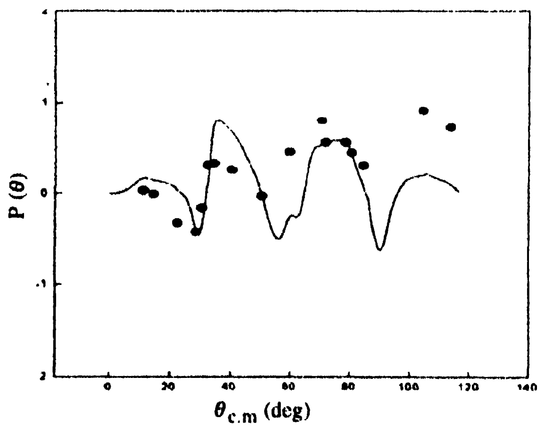

Figure 4a. Optical model fits compared to measured polarization for $^{12}C(p,p)$ at E_p = 40 MeV.

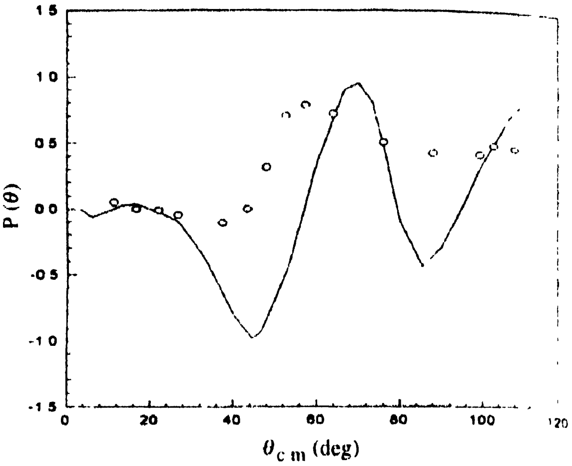

Figure 4b. Optical model fits compared to measured polarization for $^{40}Ca(p,p)$ at E_p = 40 MeV

The optical model fits satisfactorily for lower angles. It may be concluded here that the standard optical model formalism can adequately reproduce the scattering phenomena for the nuclei and the range of energies considered. It is felt that good fits to the higher energies can be obtained using a mixture of surface and volume absorptions.

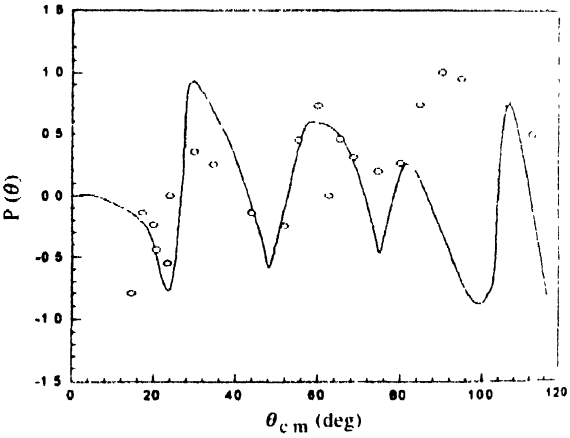

Figure 5a. Optical model fits compared to measured polarizations for $^{58}Ni(p,p)$ at E_p = 39.6 MeV

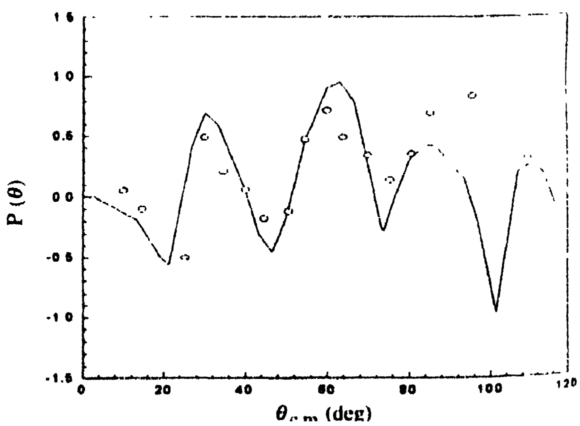

Figure 5b. Optical model fits compared to measured polarizations for $^{58}Ni(p,p)$ at E_p = 40 MeV.

Bouyssy *et al* [11a] have mentioned that at 10 MeV, the potential has a pure surface part, while we get more and more absorption in the interior when the incident energy increases. Difficulties were experienced in the simultaneous fitting of differential cross sections and polarization data. It is clear for the polarization of protons from ^{12}C, only the general feature of the polarization is grossly reproduced. On the other hand for ^{40}Ca, the fit is excellent upto about 90° and at this angle, the position of a maximum is only reproduced.

The polarization data of protons from other target nuclei such as $^{60,62,64}Ni$, $^{63,65}Cu$, $^{64,66,68}Zn$ show an excellent fit upto 80°, beyond which the fit is poor. For ^{208}Pb and ^{90}Zr, the quality of fitting is good upto 95° and above this angle, the quality of fits is not much acceptable. We have observed that improved fits could be obtained by fitting the polarization data only, resulting in a smaller spin-orbit diffuseness parameter. During the analysis of polarization data, it was found that the present optical model analysis could explain polarization phenomenon adequately upto around 85° only. This is support of the strong absorption model due to Frahn and Venter [12]. From an analysis of the potential parameters obtained (Tables 1 and 2) in the work, following trends were observed.

Table 1. Optical model potential parameters for the nuclei ^{27}Al, ^{28}Si, ^{64}Cu, ^{108}Ag, ^{197}Au (p, p) at 28 MeV

Nucleus	E_p (MeV)		U (MeV)	r_U (F)	a_U (F)	W (MeV)	r_W (F)	a_W (F)	V_{so} (MeV)	r_{so} (F)	a_{so} (F)	r_c (coulomb radius parameter)
^{16}O	44.1	(a')	38.53	1.17	0.75	21.37	1.32		6.20	1.01	0.75	1.25
		(b)	35.61	1.09	0.75	7.70	1.50	0.33	3.61	1.08	0.52	1.25
^{27}Al	28	(a)	47.10	1.21	0.71	5.80	1.27		8.50			1.25
		(b)	46.55	1.21	0.71	13.54	1.27	0.29	8.50	1.21	0.71	1.25
^{28}Si	28	(a)	59.60	1.06	0.78	0.68	0.91		2.80			1.25
		(b)	53.88	1.06	0.78	7.83	0.91	1.01	2.80	1.06	0.78	1.25
^{64}Cu	28	(a)	55.70	1.1	0.74		1.16		6.50			1.25
		(b)	43.11	1.3	0.58	9.78	1.15	0.61	1.96	1.30	0.58	1.25
^{108}Ag	28	(a)	44.90	1.17	0.57	4.40	1.11		11.90			1.25
		(b)	47.01	1.27	0.57	9.10	1.11	0.61	11.90	1.27	0.57	1.25
^{197}Au	28	(a)	50.00	1.24	0.85	5.30	1.52		8.60			1.25
		(b)	47.04	1.24	0.55	5.85	1.52	0.91	8.60	1.24	0.55	1.25

(a') Ref [8]; (a) Ref [13], (b) Present work

Table 2. Optical model potential parameters for the nuclei $^{63,65}Cu$, $^{58,60,62,64}Ni$, $^{64,66,68}Zn$, at 30 and 39.6 MeV

Nucleus	E_p (MeV)		U (MeV)	r_U (F)	a_U (F)	W (MeV)	r_W (F)	a_W (F)	V_{so} (MeV)	r_{so} (F)	a_{so} (F)	r_c (coulomb radius parameter)
^{63}Cu	30	(a)	57.87	1.12	0.77	3.45	1.37	0.64	6.90	0.02	0.58	1.20
		(b)	49.68	1.13	0.76	8.71	1.35	0.63	6.29	1.08	0.73	1.20
^{65}Cu	30	(a)	53.71	1.11	0.79	3.54	1.34	0.65	6.30	1.03	0.56	1.20
		(b)	49.74	1.13	0.78	9.60	1.35	0.62	6.20	1.11	0.71	1.20
^{58}Ni	39.6	(a)	48.20	1.27	0.46	5.20	1.27	0.46	6.65	1.17	0.63	1.20
		(b)	43.66	1.21	0.70	8.26	1.24	0.50	6.50	1.09	0.66	1.20
^{60}Ni	39.6	(a)	49.70	1.27	0.48	5.20	1.27	0.48	6.28	1.16	0.64	1.20
		(b)	44.50	1.20	0.70	7.52	1.25	0.52	6.17	1.08	0.67	1.20
^{62}Ni	39.6	(a)	46.00	1.29	0.56	4.57	1.29	0.56	6.29	1.2[illegible]	0.60	1.20
		(b)	44.41	1.19	0.70	7.01	1.29	0.63	6.35	1.04	0.71	1.20
^{64}Ni	39.6	(a)	48.90	1.26	0.60	5.60	1.26	0.60	6.78	1.18	0.60	1.20
		(b)	47.47	1.16	0.71	6.76	1.29	0.67	7.88	0.94	0.85	1.20
^{64}Zn	39.6	(a)	44.80	1.25	0.61	4.54	1.25	0.62	6.12	1.20	0.61	1.20
		(b)	45.68	1.16	0.73	6.15	1.30	0.65	6.15	1.01	0.71	1.20
^{66}Zn	39.6	(a)	43.16	1.27	0.63	4.48	1.27	0.63	6.00	1.23	0.60	1.20
		(b)	47.88	1.17	0.70	6.08	1.29	0.72	7.98	0.87	0.95	1.20
^{67}Zn	39.6	(a)	46.41	1.23	0.66	5.77	1.23	0.66	6.24	1.20	0.61	1.20
		(b)	45.60	1.15	0.73	4.87	1.30	0.74	6.78	0.94	0.86	1.20

(a) Ref. [14] for 40 MeV and [15] and [16] for 50 MeV data. (b) Present work.

(a) The values of U increases with increasing neutron number. This tendency is in agreement with the results obtained from the analysis of proton scattering by Perey and Percy [6].

(b) The radius and diffuseness are largely independent of the incident particle energy and the target mass number.

(c) Imaginary potential depth W is decreased over the sequence of nuclei.

(d) Spin-orbit potential depth remained nearly constant during increasing mass number.

During the analysis, it is also felt that more accurate experimental data, specially on polarization is required for more elaborate and better optical model analysis.

References

[1] M Rahman, A H Khan and H M Sen Gupta *Nuovo Cim* **B44** 36 (1966)

[2] H R Sultana, A S B Tariq, A F M M Rahman, M A Uddin and A K Basak *J Nuclear Sci Applic* (2000)

[3] A S B Tariq, A F M M Rahman, S K Das, A S Mondal, M A Uddin, A K Basak, H M Sen Gupta and F B Malik *Phys Rev* **C59** 2558 (1999)

[4] S K Das, A S B Tariq, A F M M Rahman, P K Roy, M N Huda, A S Mondal, A K Basak, H M Sen Gupta and F B Malik *Phys Rev.* **C60** 044617 (1999)

[5] E E Gross, R H Bassel, L N Blumberg, B J Morton, A Van der Woude and A Zucker *Nucl. Phys.* **A120** 673 (1967)

[6] C M Perey and F G Percy *Phys Rev.* **132** 755 (1963)

[7] P E Hodgson *Nuclear Reactions and Nuclear Structure* (Oxford Clarendon) Ch. 6-8 (1971)

[8] F D Becchetti and G W Greenlees *Phys. Rev* **182** 1190 (1969)

[9] F A Brieva and J R Rook *Nucl. Phys.* **A291** 299 (1977)

[10] R C Barrett, A D Hill and P E Hodgson *Nucl Phys* **62** [illegible] (1965)

[11a] A Bouyssy, H NGO and N Vinh Mau *Nucl. Phys* **A371** [illegible] (1981)

[12] W E Frahn and R H Venter *Ann. Phys.* **24** 243 (1963)

[13] S Harbhajan, Sandhu, K Robert Cole and N Charles Waddell *Nucl Phys* **A126** 594 (1969)

[14] L N Blumberg, E E Gross, A Van der Woude, A Zucker and R H Rassel *Phys. Rev* **147** 817 (1966)

[15] G L Thomas, E J Burge and D A Smith *Nucl. Phys* **A171** [illegible] (1971)

[16] J A Fannon, E J Burge, D A Smith and N K Ganguly *Nucl Phys* **A97** 263 (1967)